



National Aeronautics and
Space Administration

OH, WHAT A PANE!

An inquiry based activity with a mathematical
approach to investigating windows on
Earth....and in space.

TEACHER GUIDE



ARES

Astromaterials Research & Exploration Science



National Aeronautics and
Space Administration

OH, WHAT A PANE!

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OH, WHAT A PANE!

5-E Activity – Teacher’s Guide

Goal: Students will use mathematics to investigate windows and astronaut photographs.

Objectives: Students will:

- Identify shapes of windows and their functions.
- Investigate and calculate area, cost, and cost analysis for various window shapes.
- Analyze data from NASA astronaut photography.
- Interpret lens size and area of coverage information.
- Mathematically describe astronaut photographs.

Grade Level: 5 – 10 (Unit begins at a fifth grade level and the mathematics becomes progressively more difficult ending at the tenth grade level.)

Time Requirements: Unit does not have to be completed all at once. Teachers can be flexible with the time and use of the unit. Estimated total time of entire unit: 3 – 4 hours.

Materials:

- Student Guide Booklet
- Pattern Blocks or Tangrams
- *Oh, What A Vision!* handout
- *Oh, What A Cupola!* handout (optional extension)

National Mathematics Standards (NCTM) Addressed:

Number and Operations:

- Understand numbers, ways of representing numbers, and relationships among numbers.
- Compute fluently.

Geometry:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement:

- Apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis:

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
- Develop and evaluate inferences and predictions that are based on data.

Connections:

- Recognize and apply mathematics in contexts outside of mathematics.



Common Core State Mathematics Standards (CCSS) Addressed:

Grade 5

Number and Operations in Base Ten

- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Measurement and Data

- Convert like measurement units within a given measurement system.

Geometry

- Classify two-dimensional figures into categories based on their properties.

Grade 6

The Number System

- Compute fluently with multi-digit numbers.

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

Grade 7

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Geometry

- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Grade 8

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Geometry

- Understand and apply the Pythagorean Theorem.

High School

Geometry

- Make geometric constructions.
- Apply geometric concepts in modeling situations.

Statistics and Probability

- Making inferences and justifying conclusions.

Useful Websites for Additional Background Knowledge:

- NASA Gateway to Astronaut Photography homepage: <http://eol.jsc.nasa.gov>
- NASA Earth Observatory webpage with featured articles, images, news, and global maps of Earth: <http://earthobservatory.nasa.gov/>
- NASA ARES Expedition Earth and Beyond webpage: <http://ares.jsc.nasa.gov/ares/eeab/index.cfm>
- The Art Glass Association: <http://www.thestorefinder.com/glass/library/history.html>



- Focal Length Information: <http://www.photoaxe.com/understanding-the-lens-focal-length-and-aperture>

References, Resources and Acknowledgements:

- Runco, M., Eppler, D., Scott, K.P., and Runco, S. *Earth Science and Remote Sensing Capabilities of the International Space Station: The Destiny Module Science Window and the Window Observational Research Facility.*
- Elm City Photo, Waterville, Maine
- Hammond Lumber Company, Belgrade, Maine

Printing Alternative: As your resources permit, you can download the pdf of the Student Guide on your student computers and have students fill in answers to questions, save their work, and continue each day without printing anything. You can use Adobe Reader (<http://get.adobe.com/reader/>) or FoxIt Reader (<http://www.foxitsoftware.com/pdf/reader>). Both are freely available. Be sure to have students test this before completing the entire activity.

Adaptations, Extensions, and Enrichment:

Oh, What Do You See?

- If there is not a lot of variety in windows in your locale, bring in pictures from magazines. You could also use the Internet to find pictures to have available.
- Traditionally windows in the US are sold in customary units. You might want to rework the initial part of this activity with dimensions for the windows being given in metric units.

Oh, But the Cost!

- Explore the costs of windows in your community. Visit a local business and compare the prices of windows. If you don't have that opportunity, you can explore pricing on-line.
- Students can create paper/tissue paper stained glass windows. Have them find the area of each color.
- Assign a cost per square unit, and have students calculate the cost of each color.

Oh, What a View!

- Visit the Gateway to Astronaut Photography of Earth website (<http://eol.jsc.nasa.gov>). Have students explore the website and discuss ways in which mathematical information is used.
- Have students conduct a mathematical investigation using [astronaut photographs](#). As part of the Expedition Earth and Beyond program, students can conduct an investigation and request a new astronaut photograph as part of their investigation. For more information check out the Expedition Earth and Beyond website (<http://ares.jsc.nasa.gov/ares/eeab/index.cfm>)

Oh, What a Cupola! – OPTIONAL ACTIVITY INCLUDED AT THE END OF THE TEACHER GUIDE

- Using Google Earth, create a model of the Cupola.



Introduction and Background

Oh, What a Pane! offers a highly aligned mathematical supplement to the *Expedition Earth and Beyond* Program. The unit begins with mathematics concepts that fifth grade students would explore. The mathematics become progressively more difficult with each section. Towards the end of the unit concepts may challenge a more advanced learner.

The mathematical content for each section is outlined below:

- **Oh, What Do You See?** – shape recognition, data collection and analysis, patterns, area of rectangles (including squares which are special rectangles), circles, and octagons.
- **Oh, But the Cost!** – unit analysis, area, mathematical communication.
- **Oh, What a View!** – data analysis, percent, percent increase.
- **Oh, What a Vision!** – algebraic relationships.
- **Oh, What a Cupola! (extension)**– geometric models, technology.

NCTM Principles and Standards	Number and Operations	Algebra	Geometry	Measurement	Data Analysis and Probability	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Oh, What Do You See?	X			X	X	X		X	X	X
Oh, But the Cost!	X		X	X	X	X	X	X	X	X
Oh, What a View!	X	X			X	X	X	X	X	X
Oh, What a Vision!		X			X	X	X	X	X	
Oh, What a Cupola!			X	X	X	X			X	X

Common Core State Standards (Mathematics Domains)	Oh, What Do You See?	Oh, But the Cost!	Oh, What a View!	Oh, What a Vision!	Oh, What a Cupola!
Grade 5	5.NBT 5.MD 5.G	5.NBT 5.G	5.NBT		
Grade 6	6.NS	6.NS	6.NS		6.G
Grade 7	7.NS	7.NS 7.G	7.NS		6.G
Grade 8	8.NS 8.G				
High School				S-IC	G-CO G-MG

The mathematics content that is listed above is integrated throughout the 5-E Inquiry Model of Instruction. Each section of *Oh, What A Pane!* does not contain all parts of the 5-E model. The complete activity does incorporate all parts of the 5-E model. Each section can be given to students as an independent activity or as a total unit.



5-E INQUIRY MODEL OF INSTRUCTION

The 5-E model is an inquiry-based model of instruction based on a constructive approach to learning (learners build or construct ideas by comparing new experiences to their existing framework of knowledge). The 5-E model of instruction breaks this approach into 5 phases. The phases are: *Engagement*, *Exploration*, *Explanation*, *Elaboration*, and *Evaluation*. This model builds on prior knowledge and common experiences of students and teachers to construct or build meaning and connections to new concepts while also correcting any inaccuracies. This activity is designed as guided discovery to maintain a structure for learning for your students.

The table below breaks down each phase of the 5-E model. The table provides a general description of each phase and how the *Oh, What A Pane!* activity applies to each phase within the lesson.

5-E Phase	General Description	Oh, What A Pane!
<i>Engagement</i>	Teachers engage students using an activity, image or discussion to focus students' thinking on the learning outcomes of an activity.	Students will observe windows, tally and count the various shapes, look for the most common type, and explore the purposes of various windows (Oh, What Do You See?).
<i>Exploration</i>	Students actively explore and make discoveries using hands-on materials. Students develop concepts, processes and skills to establish an understanding of content.	Students will investigate areas of the windows, find unit costs, and use data to answer questions (Oh, What Do You See?; Oh, But the Cost!; Oh, What a View!; Oh, What A Vision!).
<i>Explanation</i>	Students communicate and explain concepts they have been exploring. Students use formal language and vocabulary associated with content.	Based on the data, students will justify what they believe to be the "perfect" window (Oh, But the Cost!). Students will explain/justify how they match lens size with astronaut photographs (Oh, What A Vision!).
<i>Elaboration</i>	Students extend conceptual understandings to new problems or experiences. Students reinforce and develop a deeper understanding of concepts and skills.	Students will create stained glass windows and change the perspective of the unit rate to mathematically describe a window (Oh, But The Cost!). Students will mathematically describe astronaut photographs (Oh, What A Vision!). Students will apply their understanding of lens size to geographic area covered to solve problems (Oh, What A Vision!).
<i>Evaluation</i>	Teachers and students assess new knowledge and understanding of key concepts.	Students will complete a variety of tables with correct mathematics. They will also answer questions that are posed throughout the unit.



ACTIVITY PROCEDURE

This activity procedure is provided as a suggested guide for the *Oh, What A Pane!* activity. This procedure includes sample answers and thumbnails of student pages for each section for your reference. The Student Guide is set up to allow a student(s) to work independently through the unit. You may decide to use the activity as a whole class activity, providing a myriad of discussion opportunities. You may have students work in pairs, small groups, or independently. Alternatively, you may consider using the activity as enrichment. Based on your student needs, feel free to provide additional instruction on finding the area of various shapes, additional work with unit cost and unit analysis, or other mathematical topics presented in the unit.

As students work through the activity, the first 2 sections (*Oh, What Do You See?* and *Oh, But The Cost!*), students will explore windows and determine the “perfect” window. As the activity progresses, students will make continual connections between mathematics, windows, and astronaut photographs.

Oh, What Do You See?

Overview of topics: Shape recognition, data collection and analysis, patterns, area.

Have students read through the introductory paragraphs on the first page of the activity. Have students use the sample shown in The Student Guide as a guide to how they should log information about windows they observe within the community. You might have students complete this tally prior to coming to class or provide pictures from magazines to have students log their observations.

The collage shows five pages from the 'Oh, What A Pane!' student guide. The first page is the introduction, which includes a table for recording window data. The second page is a data table with columns for 'Shape of Window', 'Number of Windows', 'Area of Window', and 'Percentage of Total'. The third page contains instructions for finding the area of various shapes like rectangles, circles, and triangles. The fourth page shows diagrams of windows and instructions for finding their areas. The fifth page is a section for determining the most common shape for a window, with a table for recording the results.

1. Filling in Data Table

Students will have their own answers. Rectangles, circles, semi-circles, hexagons, ovals, trapezoids, and octagons may be some of the shapes students observe. There are certainly other shapes for windows. You will need to check totals and the percentages students have calculated.

2. Based on your data, what is the most common shape for a window?

Students probably saw more rectangular windows.

3. Why do you think this is the case?

Students may mention cost, appealing shape, or popularity as reasons for the most common shapes, but other answers are acceptable.



4. Do you see any patterns in your data between the shape of the windows and what the purpose of the windows might be?

Students might find that homes have more rectangles, churches might have ovals, etc. Or they may find no patterns at all!

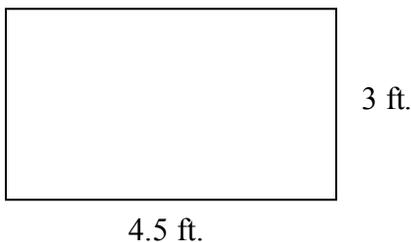
5. How is your data similar to the data of others in your class? How is it different?

Answers will vary. You might have the students discuss the amount of data collected and the location(s) the data was collected from.

One consideration you might have as you determine the “perfect” window could be the size, shape, or area of the window. You will compute the area of windows with the following shapes: rectangle, square, circle, and octagon. Be sure to show your work and final answer labeled with the appropriate unit. Use additional paper as necessary. Final answers should be recorded on the table provided in question 10.

6. Find the area of a rectangular window with dimensions 4 ½ ft. by 3 ft. Include a labeled sketch of your window and show your work.

Students should draw a rectangle with the length and width labeled. They should show the formula and calculation used to determine the area.



Example:

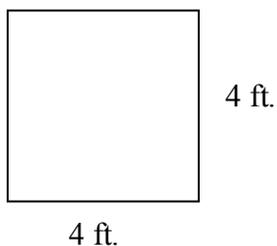
Formula For Area of a Rectangle: $A=bh$ or $A = lw$

Calculation: 4.5 (4 ½) ft. x 3 ft. = 13.5 sq. ft.

Area: 13.5 sq. ft.

7. Find the area of a square window with dimensions 4 ft. by 4 ft. Include a labeled sketch of your window and show your work.

Students should draw a square with the length and width labeled. They should show the formula and calculation used to determine the area.



Example:

Formula For Area of a Square: $A=bh$ or $A = (s)^2$

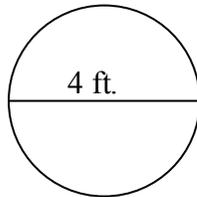
Calculation: 4 ft. x 4 ft. = 16 sq. ft.

Area: 16 sq. ft.

8. Find the area of a circular window with a diameter of 4 ft? Include a labeled sketch of your window and show your work.

Students should draw a circle with a 4 foot diameter labeled. They should show the formula and calculation used to determine the area.

Circle: Approximately 12.56 sq. ft. (using 3.14 as pi).



Example:

Formula For Area of a Circle: $A = \pi r^2$ ($\pi = 3.14$)

Calculation: Diameter = 4 ft.; Radius = 2 ft.

$3.14 \times (2)^2 = 3.14 \times 4 = 12.56$ sq. ft.

Area: ≈ 12.56 sq. ft.

Octagonal windows are a bit more difficult to calculate. According to a window manufacturer, to fit an octagonal window into a square opening of 4 ft. by 4 ft., you would have to remove $\frac{1}{4}$ in. from all sides to accommodate the frame. This would leave you with an overall height (OH) of 3 ft. 11 $\frac{1}{2}$ in. and an overall width (OW) of 3 ft. 11 $\frac{1}{2}$ in.

In order to more easily construct an octagonal window with all sides having the same length (congruent sides), this same window manufacturer stated that each side would have a length of approximately 1.6 ft. (approximately 1 ft., 7 in.).

Overall Height (OH) of the **square** opening is 4 ft. (solid black arrow). If you subtract $\frac{1}{4}$ in. from each side to fit in an octagonal window the OH would be 4 ft. minus $\frac{1}{4}$ in. from the top and $\frac{1}{4}$ in. from the bottom which equals 3 ft. 11 $\frac{1}{2}$ in. (red dashed arrow).

The Overall Width (OW) would be computed the same way.

Each side of the octagonal window is approximately 1.6 ft.

hypotenuse of approximately 1.6 ft. (According to the manufacturer of the window)

Using the Pythagorean Theorem, this side is approximately 1.13 ft.

In order to create a regular octagonal window (with congruent sides), the length of each side will be approximately 1.6 ft.

9. Let's think about this octagonal window. Looking at the sketch provided of the shaded octagon within the square, answer the following questions:

A. What do you notice mathematically about this sketch?

Students may notice what appear to be 5 squares and 4 half squares (triangles). (Other answers are possible.) If students mention the squares, the teacher may want to dispel a misconception that these are actually squares. Four of them are rectangles with one side approximately 1.6 ft. and the other side approximately 1.13 ft. and the center rectangle is the only square with dimensions 1.6 ft by 1.6 ft. (See answer 9C for an explanation of the Pythagorean Theorem.)

B. Describe how you would go about determining the area of the octagon.

Answers will vary based on how students view the above image.

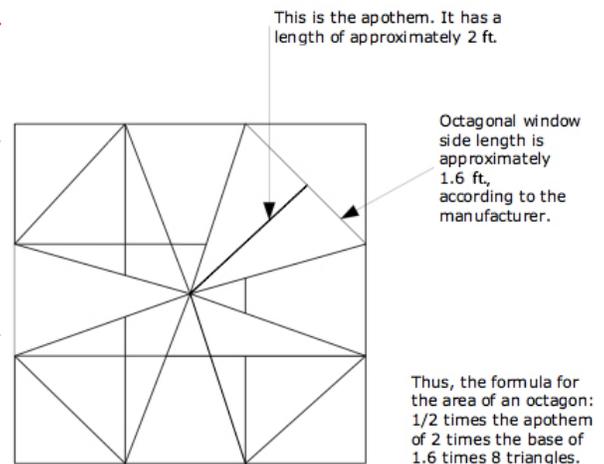


- C. Find the area of this octagonal window. Include labels on the sketch of the window provided and show your work. Use additional paper as necessary. (Students should label the sketch provided in the student guide as part of showing their work.)

Remind students that 1.6 ft. is not 1 ft. 6 in. This is another misconception. There are several methods to find the approximate area of the octagonal window. Be sure students' explanations match the method they use to solve the problem.

- *Students can use the rectangle/triangle method. The four triangles joined together can make 2 rectangles with the same dimensions as the 4 rectangles. $(1.13 * 1.6) (4 + 2) +$ the center square $(1.6 * 1.6)$ is approximately equal to **13.408 sq. ft.** They already used the Pythagorean Theorem to calculate the second dimension of each rectangle. Approximately 1.6 ft. is the hypotenuse of the isosceles triangle. $(a^2 + b^2 = c^2) \gg a^2 + b^2 = (1.6)^2 \gg a^2 + b^2 = 2.56; a = b \gg 2a^2 = 2.56$, divide both sides by 2, $a^2 = 1.28$, so a is approximately 1.13 ft.*

- *Another method would be to use the formula for finding the area of a regular polygon which is $A = \frac{1}{2}asn$, where $A =$ overall area of the polygon, $a =$ the apothem or height of the triangles that form the polygon, $s =$ the length of the sides, and $n =$ the number of sides. $A = \frac{1}{2}asn \gg A = .5(2)(1.6)(8)$. So the area is approximately **12.8 sq. ft.** (Hint: To find a (the apothem), take $\frac{1}{2}$ the distance of one side of the window casement (OH/OW). Remember that 3 ft. 11 $\frac{1}{2}$ in. divided by two is approximately 2 ft.)*



- *Students may convert all measurements that were given in feet to inches. They would need to reconvert square inches to square feet at the end of the conversion process by dividing by 144 (144 sq. in. = 1 sq. ft.; 12 in. * 12 in. = 144 sq. in.). Using $A = \frac{1}{2}asn$, students would get $\frac{1}{2} (23.75) * 8 * (1.6 \text{ ft.} * 12 \text{ in.}) = 1824 \text{ sq. in.}$ Divide by 144 and you have approximately **12.67 sq. ft.** You can use the conversion to inches for other methods as well.*
- *Another way a student might look at solving the problem is to find the total area of the square casement (4 ft. by 4 ft. = 16 sq. ft.). If students are approximating the area of the 9 sections created, they can divide 16 by 9, or multiply 1/9 by 16. Since four of the triangles represent 2 rectangles, find 7/9 of 16 sq. ft. which would be the octagon within the square window casement. This gives approximately **12.4 sq. ft.***

10. Record the area of each shape you have determined. You will use this information later.

Window Shape with Dimensions	Area (in square feet)
4 $\frac{1}{2}$ ft. by 3 ft. Rectangle	13.5 sq. ft.
4 ft. by 4 ft. Square	16 sq. ft.
Circle with 4 ft. Diameter	$\approx 12.56 \text{ sq. ft.}$
Octagon with OH/OW of 3 ft. 11 $\frac{1}{2}$ in.	$\approx 12.67 \text{ sq. ft.}$ (more than one answer is possible)



Oh, But the Cost!

Overview of topics: Unit analysis, area, mathematical communication.

For this part of the activity, students will use the area of each shape they calculated in the *Oh, What Do You See?* part of the activity. They will additionally compute costs of stained glass windows as part of their analysis of finding the “perfect” window.

Students should fill in the table as show below.

Window Type (Shape & Size)	Area in square feet	Total Cost of the Window	Work Space	Cost (in dollars) per square foot
Rectangle (4 ½ ft. by 3 ft.)	13.5 sq. ft.	\$588.00	<i>\$588 / 13.5 sq. ft. (Students should show their division.)</i>	\$43.56
Square (4 ft. by 4 ft.)	16 sq. ft.	\$465.00	<i>\$465 / 16 sq. ft. (Students should show their division.)</i>	\$29.07
Circle (4 ft. diameter)	12.56 sq. ft.	\$1,662.00	<i>\$1662 / 12.56 sq. ft. (Students should show their division.)</i>	\$132.33
Octagon (side approximately 1.6 ft. or about 1 ft. 7 inches)	12.67 sq. ft. (more than one answer is possible)	\$648.00	<i>\$648 / 12.67 sq. ft. (Students should show their division.)</i>	\$51.15

1. Use the information you have calculated in your table. Which window out of the four is the best buy? Explain how you determined this.

The best buy is the square window. You determine square footage by dividing the total cost by the area of each window.



2. Would you ever choose a window that was not the best buy, or one that was not the most cost effective? Explain.

Students may have their own answers here. They might choose a more expensive window because it is more decorative. They may want a window that is unique. They might have a specific purpose or function for the window, such as a large, picture window that would give a scenic view.

3. Find the cost per square foot of a stained glass rectangle: 30 in. wide by 48 in. high at a total cost of \$1,306.95. Include a sketch and be sure to show your work.

Students should show their labeled sketch and show their work.

Stained glass rectangle: $\$1,306.95/10$ sq. ft. is approximately $\$130.70$ per sq. ft. Conversion from square inches to square feet: $30 \times 48 = 1440$ sq. in.; 1440 sq. in./144 (sq. in. in a sq. ft.) = 10 sq. ft.

4. What would the cost per square foot be of a stained glass square: 22 in. wide by 22 in. high at a total cost of \$439.95? Include a sketch and be sure to show your work.

Students should show their labeled sketch and show their work.

Stained glass square: $\$439.95/3.36$ sq. ft. is approximately $\$130.94$ per sq. ft. [Conversion from square inches to square feet: $22 \times 22 = 484$ sq. in.; 484 sq. in./144 (sq. in. in a sq. ft.) ≈ 3.36 sq. ft.]

5. If the total cost was \$414.95 for a stained glass circle with a diameter of 30 in., what would the cost be per square foot? Include a sketch and be sure to show your work.

Students should show their labeled sketch and show their work.

Stained glass circle: $\$414.95/4.91$ sq. ft. is approximately $\$84.52$ per square foot.

[Conversion from square inches to square feet: $\pi r^2 = 3.14(15 \text{ in.})^2 = 706.5$ sq. in.; 706.5 sq. in./144 (sq. in. in a sq. ft.) ≈ 4.91 sq. ft.]

6. Find the cost per square foot of a regular stained glass octagonal window with a side length of 18.75 in. with a total cost of \$500.00. Include a sketch and be sure to show your work.

Students should show their labeled sketch and show their work.

Stained glass octagon: This problem requires the use of the Pythagorean Theorem due to the limited data provided.

- We only have the length of the side of the regular octagon. There are five squares with sides of 18.75 in.*
- The area of the corner triangles can be found once you calculate the length of the legs of the triangle using the Pythagorean Theorem. Area of the squares = $5(18.75)^2 = 1757.8$ sq. in. (approximately).*
- The length of the legs of the triangles: $a^2 + a^2 = 18.75^2$ (remember the legs are equal in length). So, $2a^2 = 18.75^2$; $2a^2 = 351.5625$; $a^2 = 351.5625/2$; $a^2 = 175.78125$; $a = 13.258$ in. (approximately).*
- (Students may discover that they do not need to find the actual length of the legs because of the relationship between the 4 triangles and the Pythagorean Theorem). $2a^2$ is equal to the area of all four triangles. Therefore you could simply add the area of the 5 squares to $2a^2$ giving you: $5(18.75)^2 + 18.75^2 = 6(18.75)^2 = 2109.375$ sq. in. Remember to divide by 144 sq. in. to convert to square feet; $2109.375/144 = 14.65$ sq. ft. To find the cost per square foot divide: $\$500/14.65$ sq. ft. = **$\$34.13$ per sq. ft. (approximately).***



- Now that you have had the opportunity to investigate windows, describe which of these would be your “perfect” window. Include the shape, dimensions, cost, and whether or not you are planning to use stained glass. Be sure to include a sketch of your window with labeled dimensions. Also describe the purpose of your “perfect” window.

Students will have their own individual descriptions of their windows.

MAKING AND MATHEMATICALLY EXPLORING YOUR OWN “STAINED-GLASS” WINDOW

- Take one piece of your window, for example an equilateral triangle in the set of pattern blocks. Let this represent one base unit. Find the area of your window, using the piece you choose as one base unit. Be sure to identify your base unit below. (Note to teacher: If students are having difficulty with this question you might ask the student how many equilateral triangles will be needed to cover a trapezoid. This will help to show if the equilateral triangle is the base unit the trapezoid has an area of 3.)

One base unit = _____

Sketch the window you created below, labeling the base unit piece. Find the area of your window. Be sure to show your work.

Area of window = _____

Answers will vary based on student window design and base unit used.

- Change your perspective by having another geometric piece be one base unit. Calculate the new area. Be sure to identify your new base unit.

One base unit = _____

Sketch the window you created below, labeling the base unit piece. Find the area of your window. Be sure to show your work.

Area of window = _____

Answers will vary based on student window design and base unit used.

- Describe your window mathematically. What shapes did you use? What percent of the total area does each shape represent of your stained glass window? What percent of the total area does each color represent of your stained glass window? What happened to the total area of your stained glass window as you changed the base unit? Include any other mathematical descriptions of your stained glass window. Remember to use proper mathematical vocabulary and good mechanics for your writing.

Window descriptions will vary based on individual student window designs. If students chose a larger piece for the base unit, their overall area would be less. If they chose a smaller piece for the base unit, their overall area would increase.



SAMPLE STAINED GLASS WINDOW EXAMPLE:



Description:

My stained glass window was designed using four pattern block shapes: 4 yellow regular hexagons, 8 red isosceles trapezoids, 10 blue parallelograms, and 4 green equilateral triangles.

Shape Percentages: Hexagons $\approx 15\%$; Triangles $\approx 15\%$; Trapezoids $\approx 31\%$; Parallelograms $\approx 39\%$.

Color Percentages: Yellow $\approx 33 \frac{1}{3} \%$; Red $\approx 33 \frac{1}{3} \%$; Blue $\approx 27 \frac{7}{9} \%$; and green is $5 \frac{5}{9} \%$

One Base Unit = the equilateral triangle

Determining the area: If the equilateral triangle is equal to one base unit, the hexagon is 6 base units, the trapezoid is 3 base units, and the parallelogram is 2 base units. To find the total area of the stained glass window, multiply the number of each type of shape by its base area. There are four hexagons that are 6 base units (total of 24 base units), 8 trapezoids multiplied by an area of 3 base units each (total of 24 base units), 10 parallelograms of 2 base units each (20 base units), and 4 equilateral triangles with a base unit each (total of 4 base units).

ONE BASE UNIT =  GREEN EQUILATERAL TRIANGLE

 1 Yellow Hexagon = 6 triangles 
4 hexagons x 6 = 24 base units

 Red Trapezoids = 3 triangles 
8 trapezoids x 3 = 24 base units

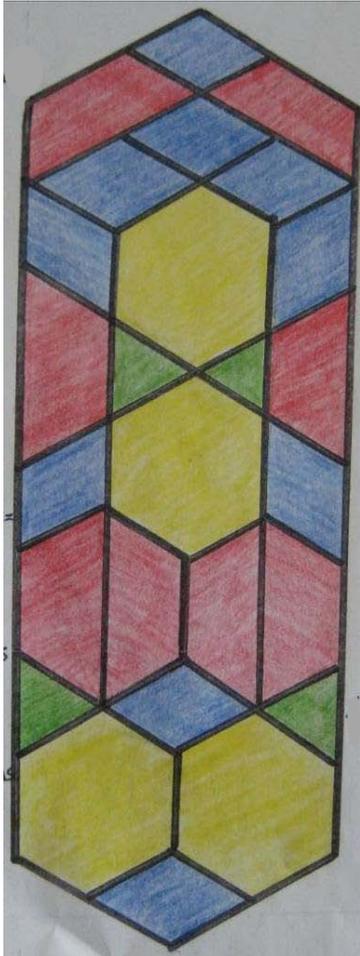
 Blue Parallelograms = 2 triangles 
10 x 2 parallelograms = 20 base units

 Green Triangles = 1 triangle 
4 triangles x 1 = 4 base units

TOTALS
24 base units (hexagons)
24 base units (trapezoids)
20 base units (parallelograms)
4 base units (triangles)

72 base units

THE ENTIRE STAINED GLASS WINDOW HAS AN AREA OF **72 BASE UNITS.**





Oh, What a View!

Overview of topics: Data analysis, percent, percent increase.

For this part of the activity, students will analyze data related to astronaut photographs taken during given time periods.

Oh, What a View!

Some windows take on a whole different perspective. Astronauts often view Earth from the "Destiny Module Science Window" when they are aboard the International Space Station. This window has the best optical quality ever placed on a human-occupied spacecraft. Astronauts take photographs, broadly referred to as "Earth Observations", that document human impacts on Earth such as city growth and agricultural expansion, natural events like hurricanes and floods, and surface features such as craters and volcanoes. Astronauts have been taking these pictures since the 1960s, forming an underlying foundation for the data collected by humans in space. Imagine how many pictures are in the NASA archives from all of their missions! Analyze the data collected from two time periods where data has been tabulated.

Data Tabulated	As of 1 April 2009	As of 26 May 2009
Number of missions in the catalog	164	166
Total images taken by astronauts	660,456	671,445
Total number of images taken from the International Space Station	354,852	365,169
Total number of images taken from the Space Shuttle	287,116	287,788

Answer each of the following questions using the data in the table above. Be sure to show your work.

1. What percentage of the total images taken through 1 April 2009 was taken from the Station?
2. What percentage of the total images taken through 26 May 2009 was taken from the Shuttle?
3. Look at the increased number of total images taken by astronaut from 1 April 2009 to 26 May 2009. What is the percent of increase?

Expedition Earth and Beyond: Astromaterials Research and Exploration Science (ARES) Education – Version 2.0
NASA Johnson Space Center

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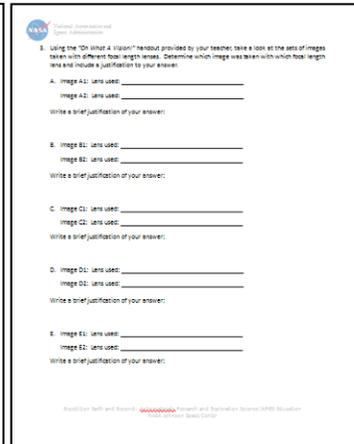
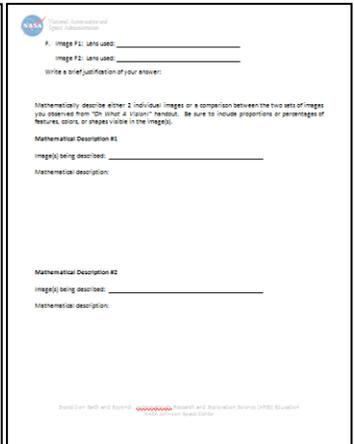
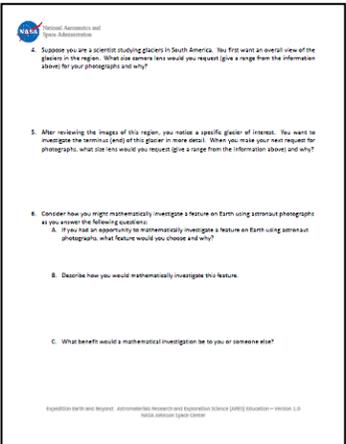
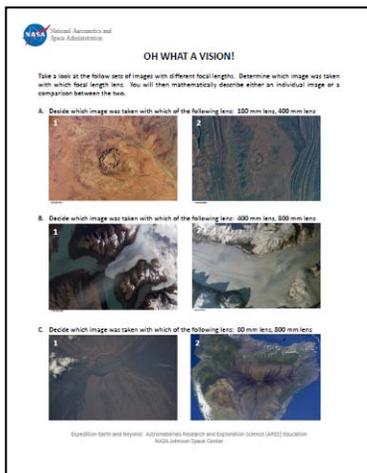
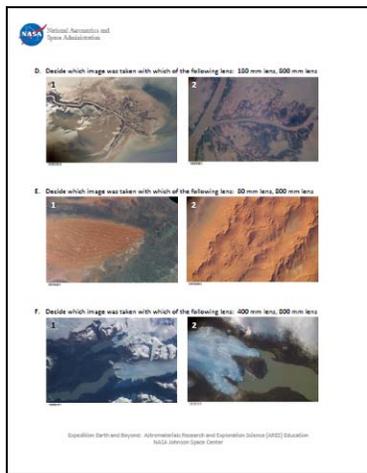
1. What percentage of the total images taken through 1 April 2009 was taken from the Station?
 $354,852 / 660,456 = \text{Approximately } 53.7\%$
2. What percentage of the total images taken through 26 May 2009 was taken from the Shuttle?
 $287,788 / 671,445 = \text{Approximately } 42.9\%$
3. Look at the increased number of total images taken by astronaut from 1 April 2009 to 26 May 2009. What is the percent of increase?
 $(671,445 - 660,456) / 660,456 = \text{Approximately } 1.66\% \text{ increase}$

Oh, What a Vision!

Overview of topics: Algebraic relationships, application of lens size/area relationship to solve scientific problems.

For this part of the activity, students will analyze the inverse relationship between lens size and area covered in an image. Students will apply their understanding of lens size to geographic area covered to solve scientific problems.

RESOURCE NEEDED: *Oh, What A Vision!* handout

These two pages are the "Oh, What A Vision!" handout used to answer questions 3 – 6.

- Based on the information above, what type of mathematical relationship explains the lens size and area of coverage?
An inverse variation
- The International Space Station's (ISS) inclination (or angle) of orbit was increased from 28.5 degrees to 51.6 degrees, significantly increasing the area of the Earth that would be visible to the astronauts through the Destiny Window. What type of mathematical relationship does this describe?
A direct variation



3. Using the handout provided by your teacher, take a look at the sets of images taken with camera lenses of different focal lengths. Match the camera lens with the acquired astronaut photograph. Include a justification of your answer.

A. Image A1: Lens used: 400 mm

Image A2: Lens used: 180 mm

Write a brief justification of your answer:

Answers will vary but should include details regarding aspects such as: the higher the focal length/lens, the less area shown in the image; or the higher the focal length/lens, the more detail is shown in the image (or vice versa).

B. Image B1: Lens used: 400 mm

Image B2: Lens used: 800 mm

Write a brief justification of your answer:

Answers will vary but should include details regarding aspects such as: the higher the focal length/lens, the less area shown in the image; or the higher the focal length/lens, the more detail is shown in the image (or vice versa).

C. Image C1: Lens used: 800 mm

Image C2: Lens used: 80 mm

Write a brief justification of your answer:

Answers will vary but should include details regarding aspects such as: the higher the focal length/lens, the less area shown in the image; or the higher the focal length/lens, the more detail is shown in the image (or vice versa).

D. Image D1: Lens used: 180 mm

Image D2: Lens used: 800 mm

Write a brief justification of your answer:

Answers will vary but should include details regarding aspects such as: the higher the focal length/lens, the less area shown in the image; or the higher the focal length/lens, the more detail is shown in the image (or vice versa).

E. Image E1: Lens used: 80 mm

Image E2: Lens used: 800 mm

Write a brief justification of your answer:



Answers will vary but should include details regarding aspects such as: the higher the focal length/lens, the less area shown in the image; or the higher the focal length/lens, the more detail is shown in the image (or vice versa).

F. Image F1: Lens used: 400 mm

Image F2: Lens used: 800 mm

Write a brief justification of your answer:

Answers will vary but should include details regarding aspects such as: the higher the focal length/lens, the less area shown in the image; or the higher the focal length/lens, the more detail is shown in the image (or vice versa).

Mathematically describe either 2 individual images or a comparison between the two sets of images you observed from “Oh What A Vision!” handout. Be sure to include proportions or percentages of features, colors, or shapes visible in the image(s).

Mathematical Description #1

Image(s) being described: _____

Mathematical description:

Answers will vary.

Mathematical Description #2

Image(s) being described: _____

Mathematical description:

Answers will vary.

4. Suppose you are a scientist studying glaciers in South America. You first want an overall view of the glaciers in the region. What size camera lens would you request (give a range from the information provided) for your photographs and why?

Students should indicate a request of a short (or small) lens size (between 70 mm and 180 mm) to get the largest area of coverage due to the inverse relationship of the lens size and area covered.

5. After reviewing the images of this region, you notice a specific glacier of interest. You want to investigate the terminus (end) of this glacier in more detail. When you make your next request for photographs, what size lens would you request (give a range from the information provided) and why?

Students should indicate a request of a longer (or larger) lens size (between 200 mm and 800 mm) to get the smallest area of coverage to help target the specific area of interest in greater detail, based on the inverse relationship of the lens size and the area covered.



6. Consider how you might mathematically investigate a feature on Earth using astronaut photographs as you answer the following questions:
- If you had an opportunity to mathematically investigate a feature on Earth using astronaut photographs, what feature would you choose and why?

Answers will vary.

- Describe how you would mathematically investigate this feature.

Answers will vary.

- What benefit would a mathematical investigation be to you or someone else?

Answers will vary.

Teacher Note: This last question is a perfect lead in to potentially having your students conduct research through the Expedition Earth and Beyond Program. For additional information contact Paige Graff at paige.v.graff@nasa.gov.

Oh, What a Cupola! (Optional Extension)

Overview of topics: Geometric models, technology.

For this part of the activity, students will create a model of the Cupola, a recent addition to the International Space Station using Google SketchUp. The Cupola includes one circular window and six trapezoid shaped windows.

RESOURCES NEEDED: Google SketchUp and *Oh, What a Cupola!* Handout.

Oh, What a Cupola!

According to NASA news in 2012, the crew of the International Space Station (ISS) is about to get a new "viewport." It's not what you'd expect when you think of space windows. It's a circular window that will be attached to the Trajectory Module (TM) known as Node 3. This dome, called the Cupola, will allow windows for observing Earth, and the marvelous expanse of the ISS itself. It'll be an 800-centimeter diameter, the center circular window of the Cupola will allow astronauts stunning views of Earth, panoramic views of Earth, and spectacular pictures of the cosmos. The Cupola is also intended for operational uses such as monitoring spacecraft, docking and use of the manipulator arm.



Before Node 3 arrived to be a part of the International Space Station built, engineers created a 3D scale model from a blueprint. Using Google Earth™ (<http://www.google.com/earth/>) create a 3D scale model of the Cupola using the information below. Note: Do not include the window coverings included in the image below in your 3D design.

Cupola Design Specifications

- Overall Height: 3.5 meters
- Base Diameter: 2 meters
- Maximum Diameter: 2.9 meters
- Top Circular Window Diameter: 75.4 cm
- Trapezoid Window Dimensions:
 - o Height: 40.3 cm
 - o Short side length: 40.9 cm
 - o Long side length: 84.4 cm

Image credit: Science Learning Center

For more details on the Cupola, visit: <http://www.nasa.gov/pdf/136077main/oh-what-a-cupola-1.01>

Handout: Oh, What a Cupola!

Handout: Oh, What a Cupola!

- Program can be downloaded for free and is available for Linux or PC systems. <http://www.google.com/sketchup/>

Once you have downloaded the program, consider the following recommendations:

1. Make sure you have time to explore the program and become familiar with the tools.
2. Encourage students to create some of the trapezoid shapes.
3. Make sure students create a rectangle for their window. A suggestion is to use Architecture Design Mode (M3).
4. Make sure you're able to rotate and duplicate the model to save any additional time they may want to use.



Image credit: Science Learning Center



Oh, What A Vision!

Take a look at the following sets of astronaut photographs. Each image was taken of the same area with camera lenses of different focal lengths. Your task is to match the camera lens with the acquired astronaut photograph.

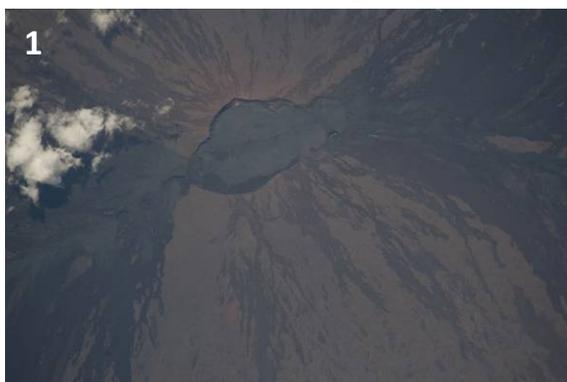
A. Match each of the following lenses with the images below: **180 mm lens, 400 mm lens**



B. Match each of the following lenses with the images below: **400 mm lens, 800 mm lens**



C. Match each of the following lenses with the images below: **80 mm lens, 800 mm lens**

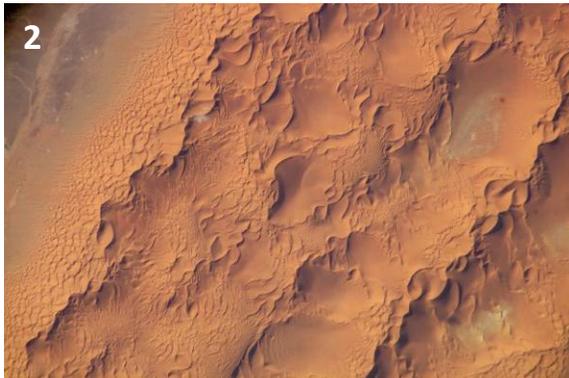
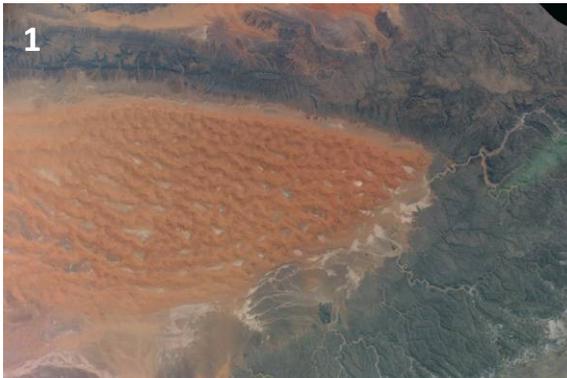




D. Match each of the following lenses with the images below: **180 mm lens, 800 mm lens**



E. Match each of the following lenses with the images below: **80 mm lens, 800 mm lens**



F. Match each of the following lenses with the images below: **400 mm lens, 800 mm lens**





Oh, What a Cupola!

According to NASA news in 2010, “The crew of the International Space Station (ISS) is about to get a new ‘eye-pod.’” A dome unlike any other window ever flown in space was launched on February 8, 2010 and attached to the Tranquility Module (also known as Node 3). This dome, called the Cupola, has seven windows for observing Earth, space, and the marvelous expanse of the ISS itself.” With an approximately 70 centimeter diameter, the center circular window of the Cupola will allow astronauts stunning views of Earth processes, panoramic views of Earth, and spectacular pictures of the cosmos. The Cupola is also intended for operational purposes such as monitoring spacecraft docking as well as the use of the manipulator arm.



Image courtesy of NASA

Before NASA contracts to have a piece of the International Space Station built, engineers create a 3D scale model from a blueprint. Using Google SketchUp (<http://sketchup.google.com/download/>), you too will create a 3D scale model of the Cupola. Note: Do not include the window coverings included in the image below in your 3D design.



Image Credit: European Space Agency (ESA)

Cupola Design Specifications

- Overall Height: 1.5 meters
- Base Diameter: 2 meters
- Maximum Diameter: 2.9 meters
- Top Circular Window Diameter: 70.6 cm
- Trapezoid-Shaped Windows:
 - Height = 40.5 cm
 - Short side length: 40.0 cm
 - Long side length: 64.4 cm

For more details on the Cupola, check out:

http://esamultimedia.esa.int/docs/hsf_research/Climate_change_ISS_presentations/Cupola_Deloo.pdf



Hints when using Google SketchUp

- Program can be downloaded for free and is available for Macs or PC's platforms. <http://sketchup.google.com/download/>
- Once you have downloaded the program, consider the following recommendations:
 1. Allow students some time to explore the program and become familiar with the tools.
 2. Encourage students to watch some of the training videos.
 3. Have students choose a template for their model. A suggestion is to use: Architectural Design Millimeters.
 4. Have students go to View, and customize the toolbar to add any additional tools they may wish to use.

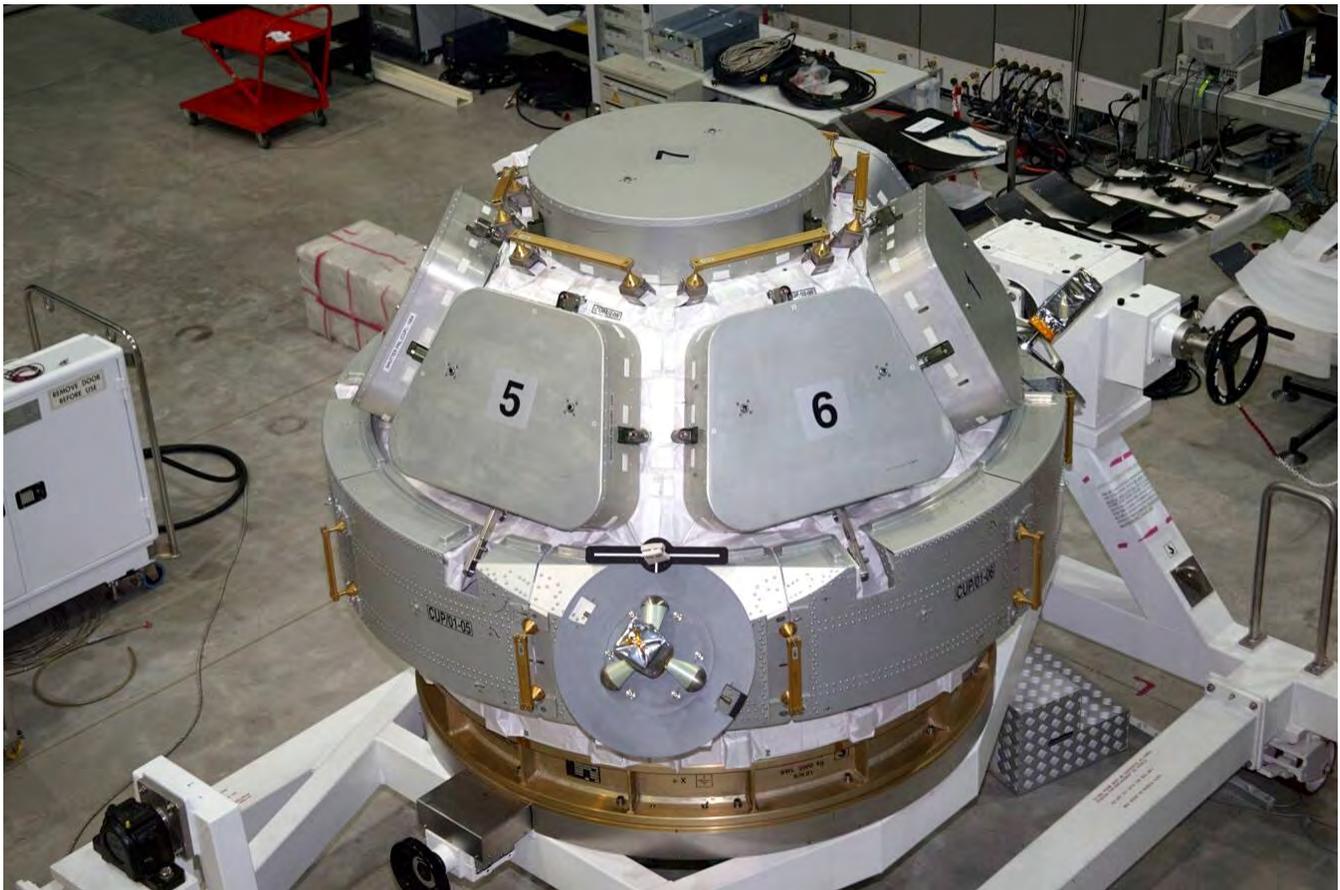


Image Credit: European Space Agency (ESA)